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(оригинал статьи на укр. языке)

На примерах показано применение полигональной регрессии для решения ряда задач в экономических исследованиях: определение точки изменения фазы жизненного цикла товара, определение размера лага между зависимыми экономическими процессами

Кл. сл.
Полигональная регрессия, экономические исследования, лаг между различными экономическими процессами, точки изменения фазы жизненного цикла

Polygonal Regression, Economic Research , the point of phases variation for the life cycle, determining of lag between different economic process

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The Use of Polygonal Regression in Economic Research

The Polygonal regression is a analytical representation of experimental data with broken line that consists of pieces of straight lines constructed with Least Squares Approach (LSE). Seber [1] considered two-phase linear regression with switching point. This regression was constructed for the cases when some point of the process is influence by some disturbance what result in abrupt change of relationship. Later on the literature the cases had been considered that included more than two phases and some efforts were made to generalize the problem to multidimensional case [2, 4]. Rastrigin [3] showed that to solve the problem of extrapolation polygonal regression is more preferable then polynomial. In Kuzmin [5] a function was suggested that allowed to describe single-dimensional multiphase regression without the system of conditions and dummy variables in the form of an analytical expression what makes much easier to perform its practical implementation.

In this work we propos to apply polynomial regression to determining the swithing point for the stages of life cycle of goods and determining the lag between several time process.

1.Determining the point of phases variation for the life cycle of goods

Life cycle of goods is one of basic theoretical statements in marketing [6] (theoretical curve of life cycle is presented in Fig.1). The action towards promotions some goods depend on the fact in which phase life cycle are the goods. However, in real life conditions the time history of sale volume has more complicated form what is explained by a large number of factors including influence of random disturbances (Fig. 2).

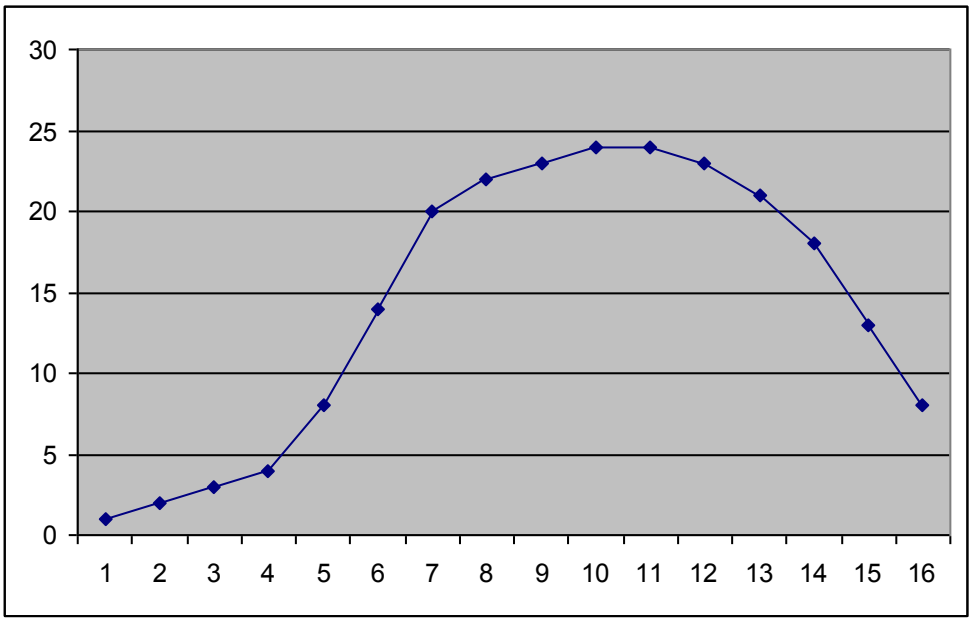

Fig.1 Theoretical curve of life cycle

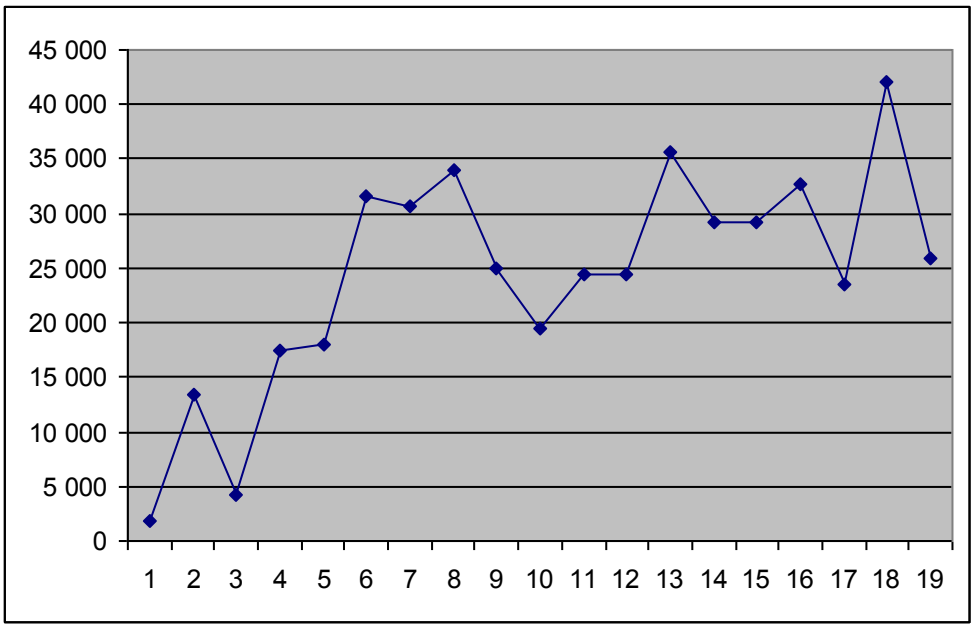

Рис. 2 Fragment of real cycle (by month in natural volume)

Evidently the line graph that reflects annual data does not exhibit fluctuations. But, taking into consideration that life cycle of goods may have only 2.5–3 years? The annual graph does not any practical sense. If we use smoothing procedure the curve will be deformed what will not provide a possibility to exactly determine the moment of transition between phases. But just this moment is important for some organization, because this information about change of life cycle phase (the moment of change) is important for making decision.

We propos to solve the problem using polygonal regression. For this purpose we suggest to use the part of a series that supposedly contains the pint of change between phases. Using the data we construct the best polygonal regression with a criterion of minimum residual variance. More complex functions than pieces of straight are not used. This explained by the fact that the residual variances cannot be distinguished statistically for a straight line and more complex functions. And, second, extrapolation using more complex function is not substantiated [3].

In the cases when the polygonal regression provides a possibility for statistically meaningful decreasing of residual regression it means that takes place change of phases of the life cycle. The point of change of phases is switching point found when the best polygonal regression is constructed.

Consider solution of the problem on the example of real data presented in Fig.2. The polygonal model has the following structure

$$Y = A + BX + C(X - X_{\Pi})_{+},$$

where X_{Π} – are coordinate of the switching point and $(X - X_{\Pi})_{+} = \dfrac{(X - X_{\Pi}) + |X - X_{\Pi}|}{2}$.

To construct optimal (using criterion of minimum residual variance $S^2_{залиш} = \sum_{i=1}^{N} (\bar{Y}_i - \hat{Y}_i)^2 \Big/ (N-k) \to \min$) polygonal regression it is necessary to find coordinates of switching point. For discrete argument the problem could solve using the method of simple search. In the case of continuous arguments it could be solved by the method of dividing a piece of line into two equal parts or random search method with a given error. Respective value of $X_П$ will be a switching point. For this example respective equation looks like follows $Y = –17774,1 + 7520,7X – 6427,3(X – 13)_+$.

In Fig.3. are show this points searched and for sake of comparison we show linear? Polygonal and logarithmic approximations.

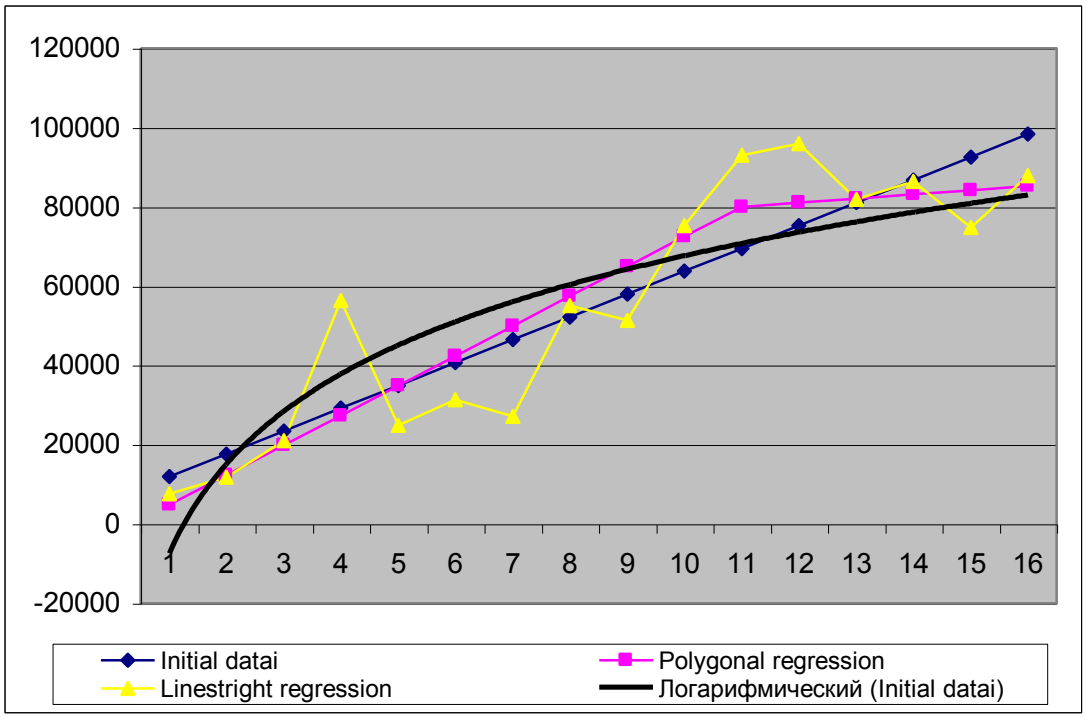

Рис. 3 Апроксимація експериментальних даних різними функціями

Comparing the three variants of approximating functions it is possible to make the following conclusions. Polygonal regression provides the smallest of residual variance. At the same time it is necessary to note that residual variance for all present functions with available substantial fluctuations on statistical data differ not more then 20–40? What relates them to the same general sample by Fisher criterion. On the other side? The use of the one function is contradicting to theoretical basis of notion of life cycle – within different phases take place different processes that must be describe by different functions. Besides? Even from the picture it could be seen that further continuation of the extrapolating lines the best prognosis will provide polygonal regression.

Thus, the use of polygonal regression allows:

- To get an approximation useful for practical application;

- To determine switching point between phases;
- To provide the best extrapolation.

The approach describes was tested on real life marketing data/ It use allows to make substantiated using statistical analysis explaining life cycle of goods.

2. Determining lag several time series

Time lag is a economic parameter that delay in time between economic processes (variables). Use the polygonal regression allows to find an estimate of lag as a difference between the points of switching for relevant processes. Consider, for example, the data taken from [7].

Table 1. Data tendency import, production and consumption of France

Year	Import	Production	Consumption
1949	15,9	149,3	108,1
1950	16,4	161,2	114,8
1951	19	171,5	123,2
1952	19,1	175,5	126,9
1953	18,8	180,8	132,1
1954	20,4	190,7	137,7
1955	22,7	202,1	146
1956	26,5	212,1	154,1
1957	28,1	226,1	162,3
1958	27,6	231,9	164,3
1959	26,3	239	167,6
1960	31,1	258	176,8
1961	33,3	269,8	186,6
1962	37	288,4	199,7
1963	43,3	304,5	213,9
1964	49	323,4	223,8
1965	50,3	336,8	232
1966	56,6	353,9	242,9

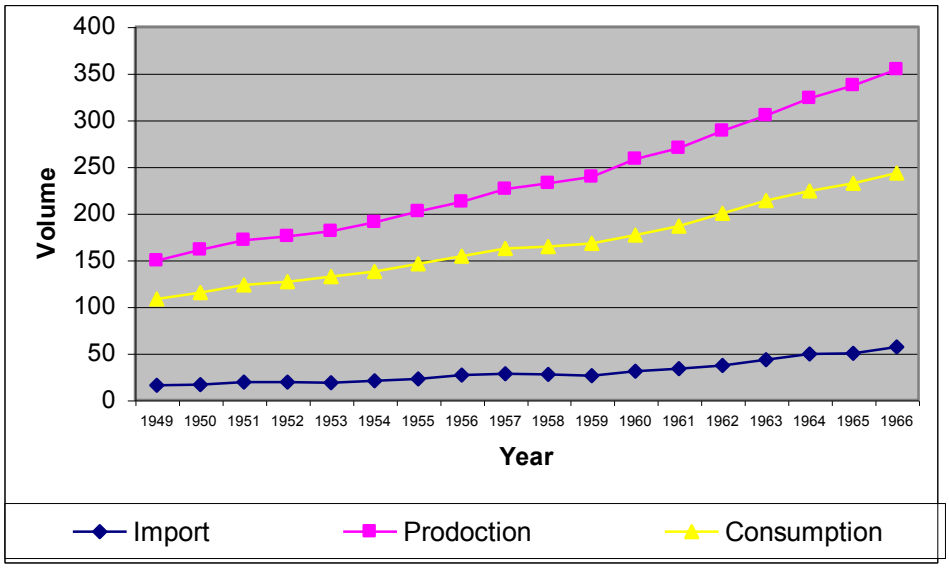

Fig.4 Data tendency import, production and consumption of France

It can be seen that due to random deviations determining of lag involves some difficulties. We constructed two-phase polygonal models for all three variables.

$Y(\text{Import}) = 14{,}06 + 1{,}30X + 3{,}09(X - 12)_+$.

$Y(\text{Production}) = 140{,}55 + 9{,}00X + 7{,}33(X - 11)_+$.

$Y(\text{Consumption}) = 102{,}49 + 6{,}21X + 5{,}02(X - 12)_+$.

The equations constructed show that after increase of consumption with a lag of one year has also increased import and production.

Conclusions

In this paper proposed new applications of polygonal regression: determining the point of switching between phases life cycle of goods and determining of lag between different economic process. The methods proposed were tested on real economic problems. To automatize the process of polygonal model construction we built macro definition in Visual Basic Application for use in electronic table Exel.

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